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**Milos Borenovic<sup>1,2</sup>**  
**Aleksandar Neskovic<sup>1</sup>**  
**Djuradj Budimir<sup>2</sup>**  
**Lara Zezelj<sup>1</sup>**

<sup>1</sup> School of Electrical Engineering, 73 Bulevar kralja Aleksandra, 11120 Belgrade, Serbia

<sup>2</sup> School of Informatics, University of Westminster

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# Utilizing Artificial Neural Networks for WLAN Positioning

Milos Borenovic, Aleksandar Neskovic, Djuradj Budimir, and Lara Zezelj

**Abstract**— Short range wireless technologies such as WLAN, Bluetooth, RFID, ultrasound and IrDA can be used to supply location information in indoor areas in which their coverage is assured. With respect to outdoor techniques, these technologies are more accurate but with smaller covering areas. In this paper, we present the comparison of the existing location techniques in WLAN networks and a novel approach of utilizing artificial neural networks for positioning purposes. In addition to estimating WLAN client's position, neural networks have been employed to estimate the room and type of the room the client resides in. Extensive measurements were conducted to evaluate these approaches and the obtained results indicate performances sufficient for real case use.

**Index Terms**—Artificial Neural Networks, positioning, radio, WLAN.

## I. INTRODUCTION

POSITIONING techniques in WLAN networks are becoming very popular. The reason for their popularity can be looked in-between the widespread of 802.11 networks and the fact that a broad scope of LBS (Location Based Services) can be brought into an existing network without the need for any additional infrastructure. There are a number of approaches to the positioning problem in WLAN networks. Unquestionably, the most popular ones are based on the Received Signal Strength Information (RSSI). Nevertheless, there are other approaches that yield additional hardware and offer superior accuracy and/or faster implementation in return.

Positioning with the use of RSSI parameter can be, in its essence, regarded as the path loss estimation problem. The nature of the path loss prediction in an indoor environment is extremely complex and dependent on a wide variety of assumptions (e.g. type of the building, construction, materials, doors, windows, etc.)[1]. Even if these basic parameters are known, precise estimation of the path loss remains a fairly complex task. Generally, statistical and deterministic

approaches can be used to address the positioning problem. The majority of techniques based on the RSSI parameter are statistical. Setting up of a statistically based positioning technique can generally be divided into two phases: off-line and on-line. In the off-line phase RSSI vectors are obtained (by measurements) at various Reference Points (RPs) and stored in the database along with the corresponding position information. In the on-line phase, the measured RSSI vector is correlated with the vectors previously stored in the database and position information is provided. This approach to determining the client location in WLAN networks is commonly referred to as Location Fingerprinting. It should be noted that the algorithm that enables the location estimate can be implemented in a broad range of manners.

This paper presents two concepts of indoor positioning based on artificial neural networks which require no detailed knowledge of the indoor environment or locations of access points.

The overview of the existing solutions for WLAN positioning is given in section II. Novel ANN based models are explained in section III. The comparison between the derived model and existing solutions is presented in section IV, whereas the conclusions are drawn in section V.

## II. EXISTING SOLUTIONS FOR WLAN POSITIONING

The prospects of using RSSI parameter for indoor positioning were first analysed in "RADAR" [2]. According to this research, it is better to use RSSI than SNR (Signal to Noise Ratio) for positioning purposes since RSSI parameter is much more dependent on the client's location than SNR. Two algorithms to establish the user location were proposed. The first one is the Nearest Neighbour (NN) algorithm which compares the RSSI vector of a mobile client against the RSSI vectors previously stored in the fingerprinting base. An extension to the proposed algorithm was also considered: the estimated location is not identified as only one RP whose RSSI vector is closest to the observed RSSI vector, but calculated as a "middle" point of  $k$  closest RPs (kNN algorithm). This analyses had shown that algorithm performance had improved for  $k = 2$  and  $k = 3$ . For larger  $k$ , this benefit was starting to fade. The other algorithm is based on simple propagation model with Rician distribution assumed. It ought to be emphasized that both approaches require minimum of three radio visible access points (APs). Measuring campaign comprised 70 RPs. At each RP measurements were made for 4

M. N. Borenović is both with the School of Electrical Engineering, 73 Bulevar kralja Aleksandra, 11120 Belgrade, Serbia and with WCRG, School of Informatics, University of Westminster, 115 New Cavendish St., London W1W 6UW, United Kingdom (e-mail: [milos@telekom.etf.bg.ac.yu](mailto:milos@telekom.etf.bg.ac.yu)).

A. M. Nešković, is with the School of Electrical Engineering; (e-mail: [neshko@etf.bg.ac.yu](mailto:neshko@etf.bg.ac.yu)).

D. Budimir is the WCRG leader, School of Informatics, University of Westminster (e-mail: [d.budimir@westminster.ac.uk](mailto:d.budimir@westminster.ac.uk)).

L. D. Zezelj is with the School of Electrical Engineering; (e-mail: [larazezelj@yahoo.com](mailto:larazezelj@yahoo.com)).

orientations of a receiver, and each measurement was averaged from 20 samples.

To produce maximum amount of information from the received RSSI vectors, the Bayesian approach was proposed [3]. This concept yields better results than the NN algorithm. The Bayes rule can be written as:

$$p(l_i | o_i) = p(o_i | l_i) p(l_i) N \quad (1)$$

where  $l_i$  is location at time  $t$ ,  $o_i$  is the observed RSSI vector at time  $t$ , while  $N$  is a normalizing factor that enables the sum of all probabilities to be equal to 1. In other words, probability that the client is at location  $l$ , if the received RSSI vector is  $o$ , is equal to the product of the probability to observe RSSI vector  $o$  at location  $l$  and probability that client can even be found at location  $l$ . The process of estimating client's location is based on calculating the conditional probability  $p(l_i | o_i)$  for each RP. The estimated client's location is equal to the RP with the greatest conditional probability. To accomplish this task, two terms on the right hand side of the (1) ought to be calculated. The first term, also referred to as the likelihood function, can be calculated based on the RSSI map (for all RP) using any approach that will yield probability density function of observation  $o$  for all RPs. As for the *a priori* probability  $p(l)$ , it ought to be calculated according to client's habits. However, for most cases the assumption of uniform distribution across all RPs is valid. The measurements were made at 70 RPs. As with the previously discussed techniques, the measurements were made for four orientations of a receiver, and each measurement was averaged from 20 samples.

Another project, named "Horus" [4], [5], had the goal of providing high positioning accuracy with low computational demands. Due to the time dependence of the signal strength from an observed AP, the authors of this project have shown that the time autocorrelation between the adjacent samples of signal strength can be as high as 0.9. To describe such behaviour, they have suggested the following autoregressive model:

$$s_i = \alpha s_{i-1} + (1 - \alpha) v_i, \quad 0 \leq \alpha \leq 1 \quad (2)$$

where  $v_i$  is the noise process and  $s_i$  is a stationary array of samples from the observed AP. Based on the model, the variance of correlated samples is equal to

$$\frac{1 + \alpha}{1 - \alpha} \sigma^2. \quad (3)$$

Throughout the off-line phase, the value of parameter  $\alpha$  is assessed at each RP and stored into database along with Gaussian distribution parameters  $\mu$  and  $\sigma$ . In the on-line phase, Gaussian distribution is modified with respect to the corresponding values of  $\alpha$  saved in fingerprinting database. Alike kNN algorithm, "Horus" system estimates the client's location as a weight centre of  $k$  RPs with the highest

probabilities  $p(i)$ . The principal difference to the kNN algorithm is that, in case of "Horus" system, the  $k$  most likely RP are pounded with their corresponding probabilities  $p(i)$ . For verification purposes, the authors made measurements on 612 RPs, and each measurement was averaged from 110 samples.

Battiti *et al.* [6] are the first to consider using ANN for positioning in WLAN networks. This approach does not insist upon detailed knowledge of indoor structure, propagation characteristics or position of APs. Multilayer feedforward network with two layers and one-step secant training function was used. The number of units in hidden layer was varied. No significant degradation in performance was noticed when the number of units grows above the optimal number. For verification purposes, measurements were made on 56 RPs, and each measurement was averaged from 100 samples.

Finally, it ought to be emphasized that averaging the RSSI vectors in the on-line phase has an immense impact on the technique's latency so the scope of location based services that could be utilized with such techniques is significantly narrowed. Moreover, bearing in mind that presented approaches require at least three radio-visible APs in each RP (which is seldom the case in most of the WLAN networks), qualitative frequency planning feasibility is uncertain and, consequently, the degradation of packet data services is inevitable when concerning positioning in larger indoor areas. Enabling the radio-visibility with 3 APs across the indoor environment is usually constructively irrational and economically unjustified. Hence, the presented techniques cannot be applied to the majority of existing WLAN networks optimized for packet data services.

### III. NOVEL APPROACH TO POSITIONING IN WLAN NETWORKS UTILIZING ANN (ETF-ANN)

Employing the ANNs for indoor positioning implies no detailed knowledge of the indoor environment or the locations of the APs. In this paper, we propose two approaches to utilizing the ANNs in positioning purposes. The first one yields coordinate estimates (ANN-A), whereas the second one provides the room number and room type estimate (ANN-B).

The layout of the building that was to be used as a test bed for this research was chosen with extreme care. The essential requirements that a chosen test bed ought to meet were: ① dimensions large enough so that the positioning technique of this class of accuracy is purposeful, ② number and deployment of the APs is optimized for Internet access, and not for the LBS, ③ complex building structure invoking a number of radio-propagation effects.

The ground floor of the Technical Schools' building at the University of Belgrade was used as a test bed. The dimensions of this floor are 147.1 m x 66.1 m, with more than 80 amphitheatres, classrooms and offices of different sizes. The rooms were numbered in circular fashion and classified with respect to propagation characteristics into the following six

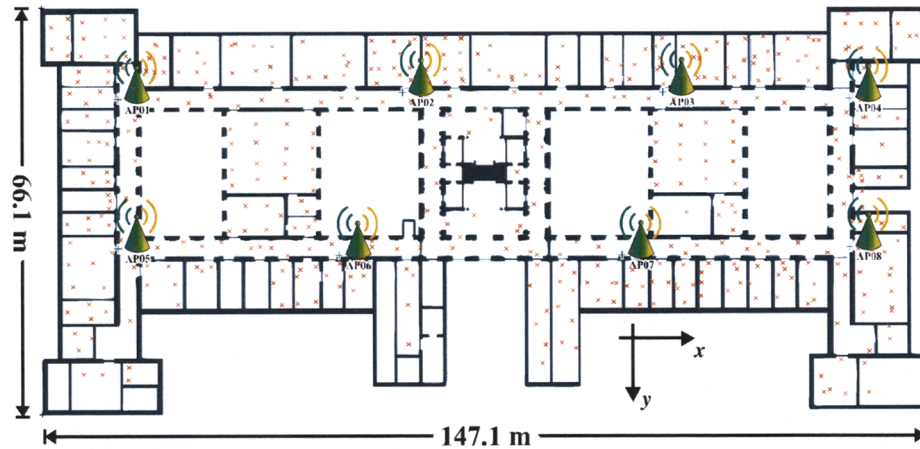


Fig. 1. The layout of the test bed, APs positions (green “+” symbols) and RP positions (red “x” symbols)

categories: long corridors, short corridors, hall, amphitheatres, classrooms (laboratories) and offices. Extended Service Set (ESS) comprised eight APs placed so as to provide optimal radio coverage for wireless Internet access.

The measuring campaign consisted of experiments performed on a number of nearly uniformly distributed RPs. Laptop computer with Cisco Aironet 802.11a/b/g Cardbus AIR-CB21AG-E-K9 wireless card and encompassing software was used as the measuring equipment. The orientation of the receiver was randomly chosen for each RP. At each RP we recorded the RSSI vector along with the position of the measuring equipment and the room number and the room type where the equipment was in. The elements of RSSI values ranged from -100 dBm to -40 dBm (dynamical range of the measuring receiver). The information that an AP is not radio-visible at a RP was coded with -105 dBm. The measuring campaign comprised 433 RPs. The layout of the test bed, as well as the positions of APs and RPs are shown in Fig. 1.

We also evaluated the measurements repetitiveness. Bearing in mind that all positioning techniques reside on the existence of the correlation between the physical values repetitively measured at the same location, it can be concluded that the measurement repetitiveness and positioning accuracy are directly related. For the purpose of quantifying measurements repetitiveness, we repeated measurements on a number of RPs. The measurements repetitiveness was evaluated through three parameters: the probability that the subset of radio-visible APs will be the same in both the first and the repeated measurement ( $P_{\Omega_1=\Omega_2}$ ), the mean absolute difference of RSSI from the observed AP ( $\eta$ ) and standard deviation of the mean absolute difference ( $\sigma$ ). Obtained repetitiveness results are shown in Table 1.

Parameter	$P_{\Omega_1=\Omega_2}$ [%]	$\eta$ [dB]	$\sigma$ [dB]
Value	20	5.12	4.13

From the results shown in Table I, it can be seen that repetitiveness is extremely low which indicates limited

positioning capabilities (regardless of the technique used). Two main reasons can account for such low repetitiveness: the complexity and dynamic behaviour of the test environment and the short time that receiver spends measuring on a channel (chosen intentionally so that the obtained results would be applicable for a broad range of wireless adapters and drivers).

With respect to the purpose that ANN is intended for and, moreover, to the nature of the problem, it has been concluded that multilayer feedforward neural networks with error backpropagation have substantial advantages in comparison to other structures [7]. The outer interfaces of the ANN must be in compliance with the number of the APs, on the input side, and with the number of values that are expected as a result on the output side of the ANN. We experimented with two different employments of the ANN. The ANN-A approach ought to provide a relative position coordinates estimate (two coordinates, since our test bed consisted of only one floor), whereas the ANN-B was to supply room and room type estimates. For both approaches, the network ought to provide two outputs and, for that reason, the chosen ANN has eight neural units (perceptrons) on the input end and two neural units on the output end.

Multilayer feedforward networks can have one or more hidden layers with perceptron units. The hidden layers with corresponding perceptron units form the inner structure of the ANN. There is no exact method for determining the optimal inner structure of the network [7]. However, there are algorithms that, starting with intentionally oversized network, reduce the number of units and converge to the optimal network structure. Nevertheless, this procedure is complex and, being that one of the main goals put in front of this technique is its simplicity, we have adopted that the first hidden layer ought to have more perceptrons than input layer so that the input information is quantified and fragmented into smaller pieces [8]. The number of perceptrons in the following hidden layers ought to decrease, converging to the number of perceptrons in the output layer. Bearing that in mind, the chosen ANN structure consists of the input layer, three hidden layers and the output layer. The number of perceptron units per layer is (from input to output) 8, 15, 9, 5 and 2.

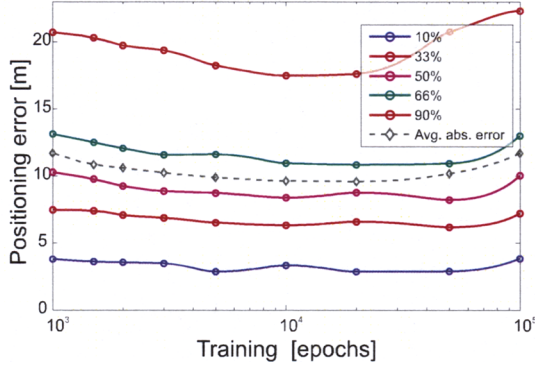


Fig. 2. Positioning error vs. training process for ANN-A approach – graph parameter are confidence percentiles (probability that a positioning error, for a given training length, is under the curve), except for grey dotted line which represents the average absolute positioning error

For the benefit of a broad scope of built-in functions for creating, initializing, training and simulation of neural networks, we have chosen to implement our ANN based model in Matlab. Both the ANN-A and ANN-B models have undergone the same training procedure with different targets as outputs. For training purposes, we have selected *traindga* training function with adaptive learning rate.

For the purpose of determining the optimal training parameters as well as the optimal training duration (in epochs), the complete set of measurements was split in two subsets containing 10% and 90% of the RPs. The larger subset was used to train the ANNs, while the smaller was used to validate the obtained model. To verify the performance of obtained model with higher accuracy this process was repeated 10 times and each time different RPs were taken for the validation purpose. This way, we obtained a verification set of 433 measurements. The performance of the ANNs were evaluated for training lengths of 1000, 1500, 2000, 3000, 5000, 10000, 20000, 50000 and 100000 epochs.

Fig. 2 shows the obtained results for the ANN-A approach. The optimal training length of the ANN-A could be defined as the “bottom” of some of the curves depicted in Fig.2. Which curve’s minimum should be concerned as the optimal training duration depends mostly on the expected set of LBS technique ought to comply with. However, for a broad range of services, it is good enough to optimize the network according to minimum of the average absolute positioning error. Fig.2. shows a minimum of the average absolute positioning error at 20000 epochs of training for which this error equals 9.58 m.

Because of the disproportion between the width and the length of the test bed, we were challenged to split the positioning error results obtained by verification to X and Y axes. For optimally trained network, the X axis median positioning error equals 5.46 m, while the Y axis median positioning error was only 3.75 m. Being that accuracy along the X axes is quite worse than along Y axes, it can be concluded that, regardless of the density of the APs, the performance of the positioning technique worsens with the increase in the dimensions of the test bed. Hence, the

performances should also be evaluated as a function of a test bed size.

The results obtained by training and validation of the ANN-B model are shown in Table II. The accuracy of the ANN-B model was evaluated through two parameters: probability that the room number ( $P_R$ ) was correctly estimated and probability that the room type was correctly estimated ( $P_{RT}$ ).

TABLE II PERFORMANCES OF ANN-B POSITIONING MODEL

Training Duration [epochs]	$P_R$ [%]	$P_{RT}$ [%]	Training Duration [epochs]	$P_R$ [%]	$P_{RT}$ [%]
1000	5.34	41.98	10000	8.40	51.66
1500	4.58	45.40	20000	8.91	55.72
2000	6.87	46.06	50000	11.5	62.34
3000	7.63	48.60	100000	11.2	62.34
5000	6.62	46.82			

Since the ANN-B approach yielded less promising results than the ANN-A model, and other presented positioning techniques do not attempt room locating, the ANN-B model will not be taken into further comparison with other presented techniques.

#### IV. COMPARATIVE ANALYSIS OF ETF-ANN AND OTHER PRESENTED POSITIONING TECHNIQUES

To compare the techniques more realistically, we introduced a derived comparison parameter that ought to normalize the positioning errors with respect to the size of the test beds where specific techniques were implemented in. For that purpose, we used the ratio between the median error and the maximum positioning error for particular test environment. Maximum positioning error is solely a function of the environment and is not technique dependent. It is calculated with the assumption of a rectangular, two-dimensional environment. It is also assumed that the position of the user is a two-dimensional, uniformly distributed random variable.

It can easily be shown that the mean absolute difference of two uniformly distributed random variables on  $[0,1]$  interval is equal to  $1/3$ . Therefore, if we denote the dimensions of test environment as  $a$  and  $b$ , maximum positioning error,  $\mathcal{E}_{\max}(a,b)$ , is given as:

$$\mathcal{E}_{\max}(a,b) = \frac{1}{3} \sqrt{a^2 + b^2}. \quad (4)$$

To correlate between the total number of the APs, the size of the test bed area and positioning accuracy, another derived parameter was used: median position error ( $\mathcal{E}_{50\%}$ ) was multiplied with the density of APs ( $\rho_{AP}$ ). From Table III, it can be seen that the best performances, regarding exclusively accuracy, are achieved by Battiti *et al.* ANN technique. The the ETF-ANN technique has somewhat worse accuracy than other presented techniques. However, the ETF-ANN technique can be successfully applied in realistic WLAN networks implemented above all for Internet access. In particular, all



other techniques require minimum of three radio-visible APs (often enough that number was significantly higher), while with ETF-ANN test bed (which is also the case with most other WLAN networks), the average number of radio-visible access points was 2.27 and in more than 50% of the RPs only one AP was “visible”. Hence, the derived parameters, that are used to give the insight to the technique’s ability to be used in a real WLAN network, should be granted considerable attention.

TABLE III COMPARATIVE ANALYSIS OF WLAN POSITIONING TECHNIQUES

Parameter	Technique	RADAR	Bayesian	Horus	Battiti ANN	ETF-ANN (ANN-A)
Test bed size – $a \times b$ [m]		43 x	25 x	59 x	20 x	144 x
x [m]		22	15	19	15	66
Covering area – $S$ [m <sup>2</sup> ]		979	375	1121	300	8500
Number of APs – $N_{AP}$		3	5	12	3	8
m <sup>2</sup> per AP – $\rho_{AP}^{-1}$		326	75	93	100	1062
Required APs number to cover ETF-ANN area size		26	113	91	85	8
RPs		70	132	110	56	433
Rx orientation at each RP		4	4	1	1	1
No. of samples for avg.		20	20	300	100	1
Number of RPs per m <sup>2</sup>		0.22	0.35	0.4	0.19	0.05
Total no. of samples / m <sup>2</sup>		60.1	28.2	120	18.7	0.05
Median error – $\epsilon_{50\%}$ [m]		3 / 4.3*	2	2	1.69	8.4
Max. error – $\epsilon_{max}$ [m]		16.3	9.72	35.8	8.33	52.8
$\epsilon_{max} / \epsilon_{50\%}$		5.4 / 3.8*	4.86	17.9	4.93	6.29
$\epsilon_{50\%} \rho_{AP}$		9.2 / 13.2*	26.7	21.5	16.9	7.91

\* fingerprinting approach /propagation model approach

When concerning the ratio between the maximum and median error, the most favourable result belongs to “Horus” technique, while the ETF-ANN falls into the second place. However, it should be pointed out that the maximum and median error ratio does not give fair assessment in case of “Horus” technique because in this case measurements were made solely in corridors with outer dimensions 59 m x 19 m while the offices surrounded by the corridors and outside of the corridors were not taken into account for positioning so the actually covered surface was significantly smaller than the one used for this comparison. Also, accuracy of the “Horus” technique benefits from time correlation of the samples, but there is an other end to such approach. Taking more time samples in the on-line phase increases the latency of the technique and narrows the potential range of the LBSs it can be used for.

The density of the APs ought not to be neglected either. Frequency spectrum is a deficient resource in WLAN infrastructural networks (2.4 GHz ISM band has a total of 14 channels of which only 3 can be non-overlapping) and, if we were to implement the existing WLAN positioning techniques in even remotely larger areas (such as the environment in

which ETF-ANN was implemented), the increased interference would induce a major setback regarding packet data services. This would have the highest impact on the ANN (Battiti *et al.*), Horus and Bayesian approach because the density of deployed APs, with these approaches, is primarily optimised towards the use of the LBS and practically hinders any efforts on frequency planning. Certainly, the increase in price of network (network devices, cabling, etc.) should be considered as well.

If we evaluate the product of median position error ( $\epsilon_{50\%}$ ) and density of the APs ( $\rho_{AP}$ ), the best performances are obtained by ETF-ANN model. Radar and ANN (Battiti *et al.*) follow, while “Horus” and Bayesian approach fall far behind.

Finally, the effort put into the off-line measurement collection should be stated as well. The total number of measurements made per square meter is by far the smallest with ETF-ANN approach (0.051 measurements per m<sup>2</sup>). All other techniques have asignificantly greater measurement density which is mostly emphasized with “Horus” technique (120 measurements per m<sup>2</sup>). The implementation of “Horus” technique in the ETF-ANN test bed would require 90 APs and more than 1000000 measurements which is, of course, very impractical.

## V. CONCLUSION

Although there is no unique and fair way to quantify and evaluate all the characteristics and performances of the aforementioned techniques, thus making the decision-making process straightforward, it has been shown that the proposed ETF-ANN (ANN-A), possibly with further modifications, presents a good choice for positioning technique in realistic WLAN networks that can qualitatively respond to the requirements of a broad scope of LBSs.

The approach concerning room number and type estimates does not yield satisfactory results for a majority of LBSs, but it should be granted further research. Future work might also include adding the SNR to the ANN inputs.

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